The Heated Round Jet in a Coflowing Stream

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Measurements are made in a heated axisymmetric turbulent jet for different values of the isothermal external stream velocity. Normalized mean velocity and temperature profiles in the jet are unaffected by the jet-to-external-stream velocity ratio and are essentially in agreement with the distributions obtained in the heated round jet into still air and with results for the heated axisymmetric wake. The turbulent temperature fluctuation intensity and the longitudinal heat flux, measured on the axis of symmetry, show a significant increase as the velocity ratio decreases. The radial turbulent heat flux and Reynolds shear stress distributions have been calculated from the mean velocity and temperature profiles and also show a marked increase with a decrease in the velocity ratio. The advection of mean squared temperature fluctuations becomes more significant vis a vis the production term when the velocity ratio decreases. Overall, the flow exhibits a behavior which lies between that for a heated jet into still air and a heated axisymmetric wake.

Nomenclature

d = nozzle diameter
f = dimensionless mean velocity profile defined by Eq. (2)
g = dimensionless mean temperature profile defined by Eq. (1)
I_n = constants, defined in Sec IV
L_c = mean concentration radius
L_θ = mean temperature radius
L_u = mean velocity radius
r = radial distance
T = mean local temperature
T_0 = value of $(T-T_1)$ on the axis
U = mean local velocity
U_0 = value of $(U-U_1)$ on the axis
u = axial velocity fluctuation
v = radial velocity fluctuation
x = axial distance
α = thermal diffusivity
η = dimensionless coordinate, defined as r/L_θ
θ = temperature fluctuation
ν = kinematic viscosity
ρ = density
σ_t = turbulent Prandtl number

Subscripts

j = value at nozzle exit
0 = value on axis of symmetry
1 = value in external stream

I. Introduction

In a previous paper,[1] the effect of a moving external stream on the form of the mean and fluctuating fields of an isothermal round jet was investigated. One of the main features of the results was that, although normalized mean velocity profiles and turbulence intensity profiles were unaffected by the velocity ratio and distance downstream, there was not universal similarity; thus values of $(\overline{u^2})_0^{1/2}/U_0$ (where u is the axial velocity fluctuation and U_0 the velocity excess on the centerline) and $(\overline{uv})_{\max}/U_0{}^2$ varied with axial distance x and with the ratio of the jet velocity to the external stream velocity. In general, the results indicated a behavior that lies between that for the classical round jet into still air and that for the axisymmetric wake. In addition, these results, together with other results for the axisymmetric wake, indicate that there is no universal self-preserving wake flow[2] and that the high ratio of advection to local production in wakes probably has a strong influence on the turbulence structure.

In the present work, the jet is heated and the effect of the external velocity on both the mean and fluctuating temperature fields is investigated. Measurements are also made for the mean and fluctuating velocity fields, and the Reynolds shear stress $\overline{uv}$ and the radial heat flux $\overline{v\theta}$ are calculatedfrom these measurements. The axial turbulent heat flux $\overline{u\theta}$ was measured directly and the advection and production terms in the budget for $\overline{\theta^2}$ determined. Results available in the literature are confined to those for a jet into still air where Wilson and Danckwerts[3] extended the early work of Corrsin and Uberoi[4] for heated jets while Uberoi and Garby[5] have studied a cooled jet. Some interesting measurements have also been made for this flow by Becker et al.,[6] who studied concentration fluctuations in a smoke-filled jet. The heated axisymmetric wake has been studied by Freymuth and Uberoi[7] and Gibson et al.[8] The present results are discussed in the context of these, which might be regarded as representing the two ends of the sepctrum of this class of flows.

II. Experimental Arrangement and Measurement Techniques

The experimental apparatus is that used for the isothermal investigation of Antonia and Bilger.[1] The jet is supplied from the laboratory high-pressure air supply and heated electrically before entering a 50-mm diameter supply pipe which contracts to the diameter (d = 15.9 mm) of the brass nozzle used in this investigation. The heater is in the form of a shell and tube heat exchanger, the tubes being insulated resistance elements with the air passing on the shell side. The spaces between the

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tubes were packed with fine nichrome wire wool to increase the heat transfer area and the elements were supplied with a dc current of approximately 30 A at a voltage of 70 V.

At the exit from the nozzle the jet temperature was maintained at approximately 170°C ($T=T_j$) above ambient temperature. The jet velocity U_j was kept constant at 45.7 msec^{-1} for all the experiments, this velocity being continuously monitored with the aid of a mercury U-tube manometer connected to an orifice plate situated in the laboratory air-supply pipe stream upstream of the heater section. The external air was supplied by a centrifugal blower and was effectively at ambient temperature (approximately 20°C). Three values of the external air velocity U_l were used for the majority of the measurements, giving values for the ratio U_j/U_l of 16.8, 5.6, and 3.0.

The mean and fluctuating temperatures were obtained with a 3-μm diameter platinum-coated tungsten wire of length 1 mm. The mean and fluctuating velocities were obtained with a 5-μm diameter wire of similar material and length as the temperature or "cold" wire. The two wires were mounted parallel to one another on the same probe at a distance of about 0.4 mm apart and were placed normal to the axial flow direction. The "velocity" or "hot" wire was operated with a DISA 55M10 constant temperature anemometer at an overheat ratio of 0.8. The "cold" wire was operated with a DISA 55M20 constant temperature anemometer at an overheat ratio of 0.8. The "cold" wire was operated with a DISA 55M20 constant current anemometer with the value of the current set at 2 mA. For this current, the velocity sensitivity of the cold wire was found to be negligible. The fluctuating voltage from the constant current anemometer was compensated for by the thermal inertia of the wire (the -3 dB point of the wire was estimated to be equal to 0.24 msec at $U=8$ msec^{-1}). The compensator was designed to give a frequency response flat up to 20 kHz. The thermal coefficient of resistivity of the "cold" wire was obtained by inserting the wire in a calibrated oven, heated for a range of temperatures corresponding approximately to the experimental conditions; it was determined to be 0.0040°C^{-1}. The mean temperature of T of the flow was estimated from the close-to-linear variation with temperature of the cold wire resistance. The mean velocity U was obtained directly from a total head tube of rectangular opening located at about 1.5 mm above the two wires with allowance made for the density variation in the jet. The velocity was also determined indirectly from the velocity calibration of the hot wire.

For small variations u in U and θ in T, the fluctuating voltages e_c and e_h for the cold and hot wires, respectively, are given approximately by

$$e_c = c\theta \qquad e_h = au + b\theta$$

where

$$a \equiv (\partial E_h/\partial U)_T \qquad b \equiv (\partial E_h/\partial T)_U$$

The coefficients a and b were at first determined from a velocity-temperature calibration of the hot wire in the potential core of the jet. This method was soon discarded as it involved a long waiting period for the temperature in the jet to reach each a new steady state (for a given velocity U_j) and led to an undesirably long time delay between calibration and experiment. The calibration method adopted was to use the actual experimental values of E_h, U, and T to determine the constants C_i ($i=0$ to 5) in the assumed equation

$$E_h{}^2 = C_0 + C_1U^{1/2} + C_2T + C_3U + C_4T^2 + C_5U^{1/2}T$$

This method is essentially similar to that described in Yeh and Van Atta[9] and has the advantage of being obtained for the relevant mean velocity and temperature range and the existing velocity and temperature fluctuation levels. The coefficients C_i depend only on the physical properties and operating conditions of the wire and were obtained by Stephenson[10] from pitot tube values of U and "cold" wire values of T by a least-squares method programmed on a CDC 6600 computer. With 56 data points, the mean error in $E_h{}^2$ was found to be 0.28% with a standard deviation of 0.28%.

The determination of $\overline{\theta^2}$, $\overline{u^2}$, and the cross-correlation $\overline{u\theta}$ followed from measured rms values of e_c, (e_c+e_h), and (e_c-e_h). The addition and subtraction operations were carried out in a DISA 55D26 signal conditioner, but prior to these operations the gains for the signals (e_c+e_h) and (e_c-e_h) were adjusted in order to improve the accuracy of measurement for $\overline{u\theta}$. To minimize the duration of an experimental run it was sometimes convenient to record e_c and e_h on a Hewlett-Packard analog FM tape recorder at a speed of 1.52 msec^{-1}, with the tape played back at a later stage for final processing of the data.

III. Mean Velocity and Temperature Fields

Mean temperature and velocity profiles across the shear flow are shown in Figs. 1 and 2 for various values of U_j/U_l and x/d. The temperature profiles are presented in the form $\Delta T/T_0$ vs η where ΔT is the difference between the local temperature and the temperature T_l of the external stream and T_0 is the value of ΔT on the axis of symmetry. The coordinate η is equal to r/L_0, where L_0 is chosen as the half temperature radius, i.e., the distance to the radial position where $\Delta T = T_0/2$. The profiles of Fig. 1 show good similarity for x/d greater than about 20 with

$$\Delta T/T_0 = g(n) \tag{1}$$

where g can be approximated by the Gaussian function $e^{-0.693\eta^2}$ when η is less than 1.5. The mean velocity profiles of Fig. 2 are narrower in extent than $\Delta T/T_0$, but also show close similarity with

$$(U-U_l)/U_0 = f(\eta) \tag{2}$$

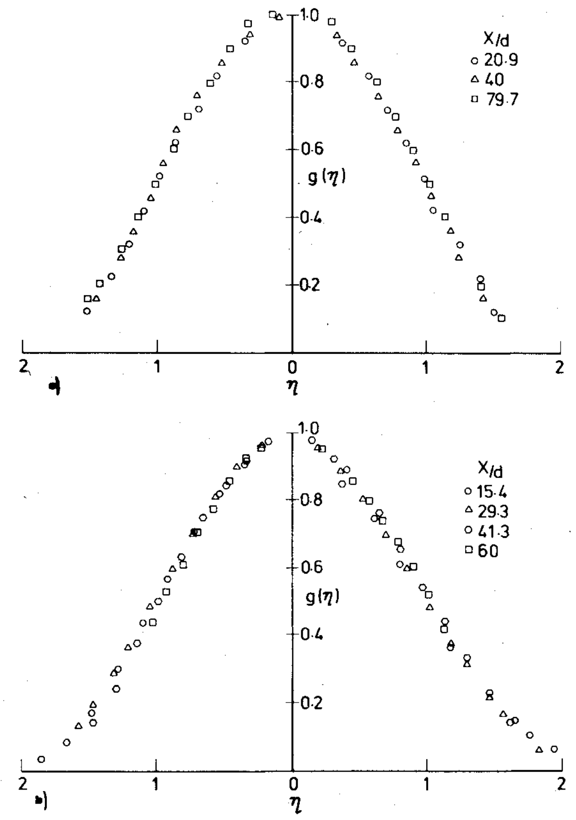

Fig. 1 Mean temperature profile for: a) $U_j/U_l = 3$; b) $U_j/U_l = 5.6$.

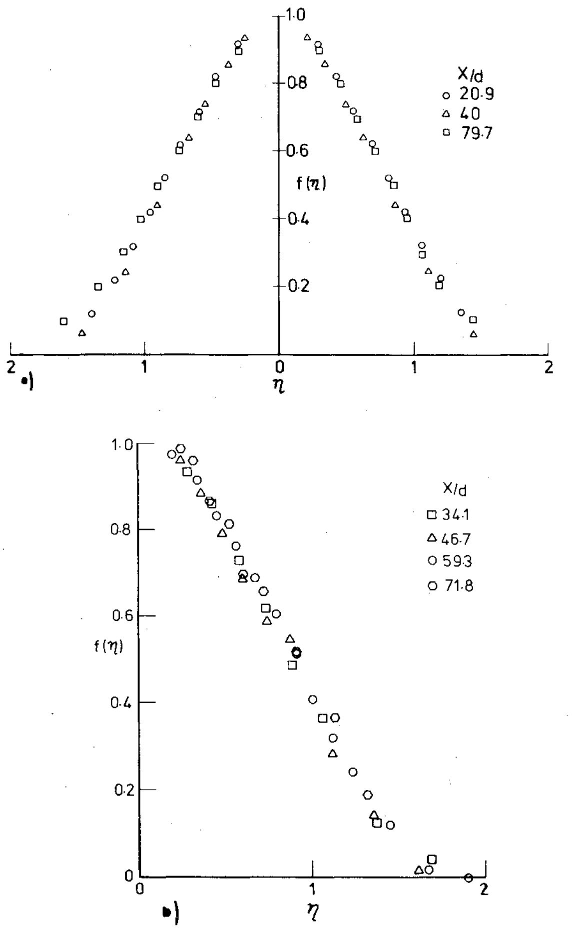

Fig. 2 Mean velocity profile for: a) $U_j/U_l=3$; b) $U_j/U_l=5.6$.

where U_0 is the value of $(U-U_l)$ measured on the axis of the flow and $f(\eta)$ is approximated by the error function $e^{-1\cdot 02\eta^2}$ for $\eta<1.2$. Both $g(\eta)$ and $f(\eta)$ are in close agreement with the temperature and velocity profiles measured by Corrsin and Uberoi[4] in the case of a heated jet with no external stream.

The distributions of T_0/T_j and U_0/U_l along the axis of symmetry are given in Fig. 3. There is a significant decrease in T_0 as U_j/U_l increases, but T_0/T_j appears to vary approximately inversely with x/d for all three values of U_j/U_l. For $U_j/U_l=16.8$ the distribution of T_0/T_j lies slightly below the measurements of Corrsin and Uberoi[4] and the correlation proposed by Wilson and Danckwerts[3] for $20°C<T_j<200°C$

$$T_j/T_0=0.175(x/d)(\rho_1/\rho_j)^{1/2} \qquad (3)$$

(ρ_l and ρ_j refer to the densities of the ambient air and the jet, respectively). At the lowest value of U_j/U_l considered here, there is no indication of the asymptotic $x^{-2/3}$ dependence of T_0 as obtained in the heated axisymmetric wakes of Freymuth and Uberoi, and Gibson et al.[8] Such a dependence would be expected from the constancy of heat flux across all planes perpendicular to the x axis, which can be expressed as

$$\int_0^\infty \rho r U \Delta T \mathrm{d}r=\mathrm{const}$$

or

$$T_0 L_0{}^2\int_0^\infty g(U_l+U_0 f)\eta \mathrm{d}\eta=\mathrm{const} \qquad (4)$$

In the case of a strong jet, $U_0>>U_l$ and it follows from Eq. (4) that $T_0\sim x^{-1}$ when $L_0\sim x$ and $U_0\sim x^{-1}$. In the case of a

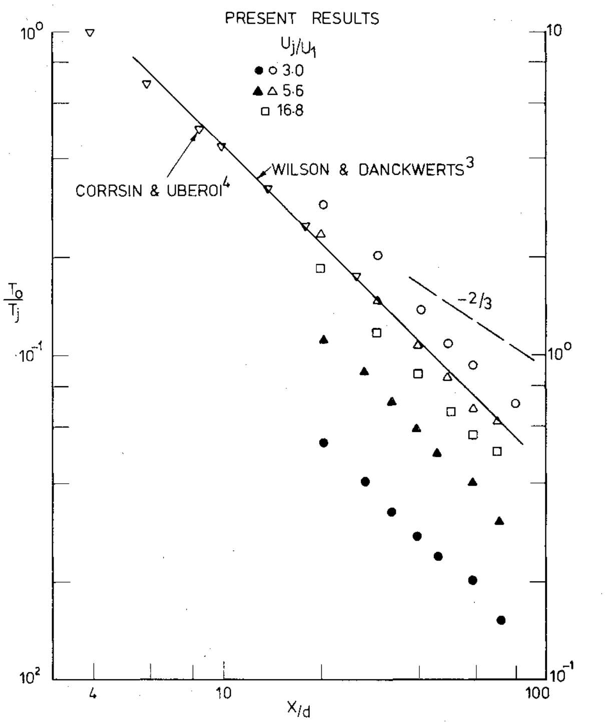

Fig. 3 Decay of excess temperature and velocity on axis of jet.

small perturbation jet or wake, $|U_0|<<U_l$, and with $L_0\sim x^{1/3}$, it follows that $T_0\sim x^{-2/3}$. The distributions of U_0/U_l vs x in Fig. 3, like those of $\Delta T/T_0$, show an x^{-1} dependence, but U_0/U_l is significantly reduced as U_j/U_l decreases. It should be noted that for the isothermal jet[1] with $U_j/U_l=3.0$, U_0/U_l also shows an x^{-1} dependence for $x/d<100$, but deviates significantly from this for larger x/d. Corrsin and Uberoi[4] found that U_0/U_j decreases with increasing temperature of the jet and this result was confirmed by the measurements of Uberoi and Garby[5] who investigated axisymmetric jets cooled below room temperature.

The growth of L_0 with x in both heated and unheated axisymmetric jets over a range of values for U_j/U_l is given in Fig. 4. The data of Corrsin and Uberoi[4] and Wilson and Danckwerts[3] for a heated jet with $U_l=0$ lie significantly above the linear growth rate of the half-velocity radius L_u reported by Wygnanski and Fiedler[11] for the isothermal jet, also with $U_l=0$. Uberoi and Garby[5] observed a decrease in the width of the temperature profile with increased cooling of the jet but this decrease is accompanied by a proportional decrease in the width of the velocity profile. The smoke concentration profiles of Becker et al.[6] yield a half-concentration radius L_c appreciably smaller than the value of L_0 for the heated jet ($U_l=0$).

The present results show the significant decrease of L_0 with decreasing U_j/U_l. At $U_j/U_l=16.8$, the rate of growth of L_0 appears to be linear for $x/d<40$ and is not significantly lower than that reported by Wilson and Danckwerts.[3] At $U_j/U_l=3.0$, L_0 remains about 20% larger than L_u, whereas Freymuth and Uberoi[7] find that $L_0\simeq L_u$ for their heated axisymmetric wake. An estimate for the representative length scale L of temperature fluctuations is obtained from the integral of autocorrelation curves of temperature fluctuations. On the axis of the jet, for $U_j/U_l=5.6$, L is equal to about $0.37L_0$ (Fig. 4). The momentum integral equation may be written as

$$L_0{}^2 U_0{}^2\int f\left(\frac{U_l}{U_0} \quad f\right)\eta \mathrm{d}\eta=\mathrm{const} \qquad (5)$$

when the variation of ρ across the jet is neglected. With $f(\eta)=e^{-k\eta^2}$, the above equation leads to

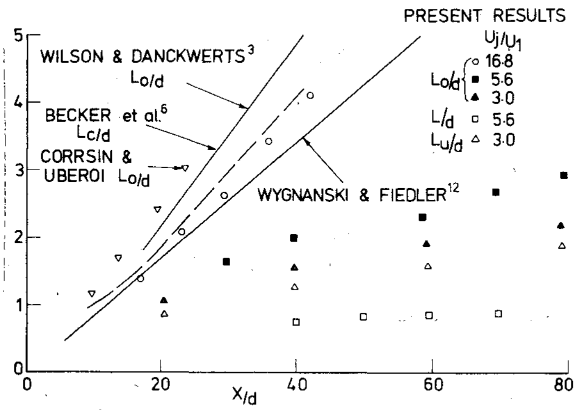

Fig. 4 Variation of length scales with x.

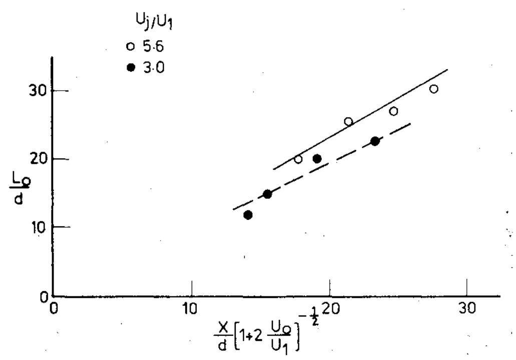

Fig. 5 Variation of L_0 as a check of momentum flux conservation.

$L_0 \sim U_0^{-1}(1+2U_1/U_0)^{-1/2}$. Since U_0 is found (Fig. 3) to vary as x^{-1}, the departure of L_0 from a linear profile can be represented by the factor $(1+2U_1/U_0)^{-1/2}$. The trend of L_0 replotted in Fig. 5 for $U_j/U_1=5.6$ and 3.0 shows that the experimental points follow closely the behavior expected from the measured U_0 and Eq. (5).

IV. Velocity and Temperature Fluctuations

The rms temperature fluctuation $(\overline{\theta^2})^{1/2}$ along the axis of the jet has been normalized by the local maximum excess temperature and shown in Fig. 6 for the x/d range covered by the experiments and for the three values of $U_j/U_1=16.8$. For $U_j/U_1=(\overline{\theta^2})^{1/2}/T_0$ appears to approach a constant value of 0.21 at large x/d, which is only slightly higher than the value of 0.18, obtained by Wilson and Danckwerts[3] for $U_j/U_1=\infty$. As U_j/U_1 decreases, $(\overline{\theta^2})^{1/2}/T_0$ shows a slight increase for $x/d>50$ and for $U_j/U_1=3.0$; $(\overline{\theta^2})^{1/2}/T_0$ is still increasing at $x/d=80$. The results of Corrsin and Uberoi,[4] which only extend to $x/d=25$ and above $x/d=10$, show a slight decrease in the streamwise direction which is at variance with the present results. The intensity of smoke concentration fluctuations $(\overline{c^2})^{1/2}/C_0$ by Becker et al.,[6] also for $U_j/U_1=\infty$, is essentially in agreement with the results of Corrsin and Uberoi for small x/d but increases for $x/d>10$. The results of Freymuth and Uberoi[12] for the two-dimensional wake of a heated cylinder indicate a value of $(\overline{\theta^2})^{1/2}/T_0$ on the wake axis of 0.21. On the other hand, Gibson et al.[8] obtained 0.38 while Freymuth and Uberoi's[7] data indicated a value as high as 0.73, both of these investigations relation to the wake of a heated sphere. These measurements tend to support the nonuniversal structure of the axisymmetric wake, discussed in Antonia and Bilger.[2]

The intensity of longitudinal velocity fluctuation $(\overline{u^2})^{1/2}/U_0$ along the axis of the jet is shown in Fig. 7. The increase in $(\overline{u^2})^{1/2}/U_0$ with decreasing U_j/U_1 is more marked than for $(\overline{\theta^2})^{1/2}/T_0$, with $(\overline{u^2})^{1/2} \simeq 0.5$ for $U_j/U_1=3.0$. For comparison, distributions of $(\overline{u^2})^{1/2}/U_0$ given in Ref. 1 for

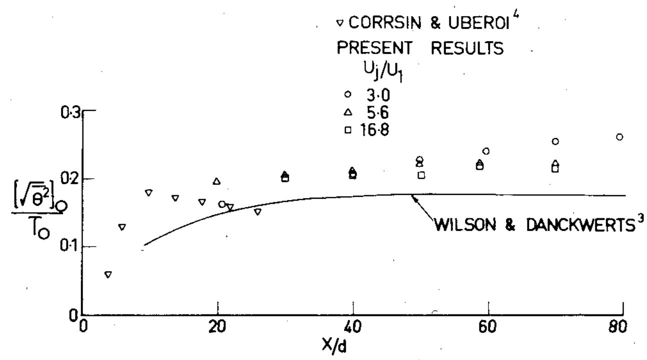

Fig. 6 Variation of temperature fluctuation intensity along jet axis.

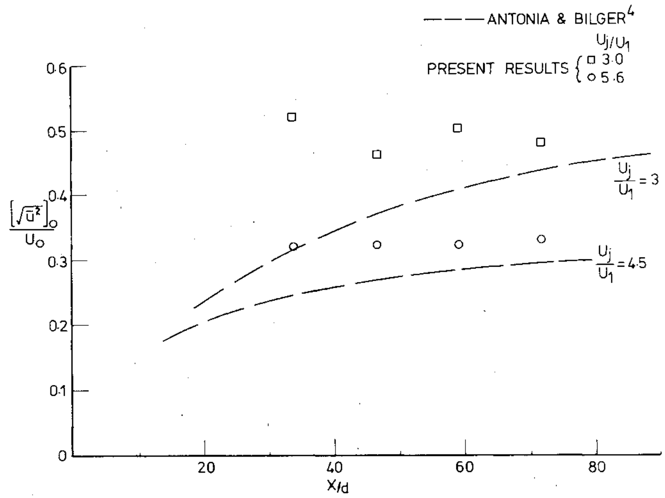

Fig. 7 Variation of velocity fluctuation intensity along jet axis.

the isothermal jet are also shown in Fig. 7 (nozzle diameter d used in Ref. 1 was equal to 5.28 mm).

Profiles of $(\overline{\theta^2})^{1/2}/(\overline{\theta^2})_0^{1/2}$ shown in Fig. 8 support, within the experimental scatter of the data, the similarity of θ fluctuations across the flow. The maximum value of $(\overline{\theta^2})^{1/2}/(\overline{\theta^2})_0^{1/2}$ obtained near $r/L_0=0.9$ is about 1.25 as compared with 1.20 (Wilson and Danckwerts[3]) and 1.12 (Freymuth and Uberoi[7]). Profiles of $(\overline{u^2})^{1/2}/(\overline{u^2})_0^{1/2}$ (Fig.9) show good similarity at different streamwise stations. The maximum value of $(\overline{u^2})^{1/2}/(\overline{u^2})_0^{1/2}$ is 1.13 and occurs near $r/L_0=0.9$. The radial extent of the u fluctuations is essentially the same as that for the θ fluctuations.

The radial turbulent heat flux $\overline{v\theta}$ has been calculated from the heat transfer equation

$$U\frac{\partial(\Delta T)}{\partial x}+V\frac{\partial(\Delta T)}{\partial r}=-r^{-1}\frac{\partial}{\partial r}(r\overline{v\theta})+\alpha r^{-1}\frac{\partial}{\partial r}\left(r\frac{\partial(\Delta T)}{\partial r}\right) \tag{6}$$

where α is the thermal diffusivity. The normal velocity V is obtained from the continuity equation

$$\partial U/\partial x+V/r+\partial V/\partial r=0 \tag{7}$$

Using Eq. (2) and integrating Eq. (7) between $r=0$ and r gives

$$V=U_0\left(\frac{dL_0}{dx}\right)[\eta f-2\eta^{-1}I_1]-L_0\left(\frac{dU_0}{dx}\right)\eta^{-1}I_1 \tag{8}$$

where

$$I_1=\int_0^\eta f\eta \, d\eta$$

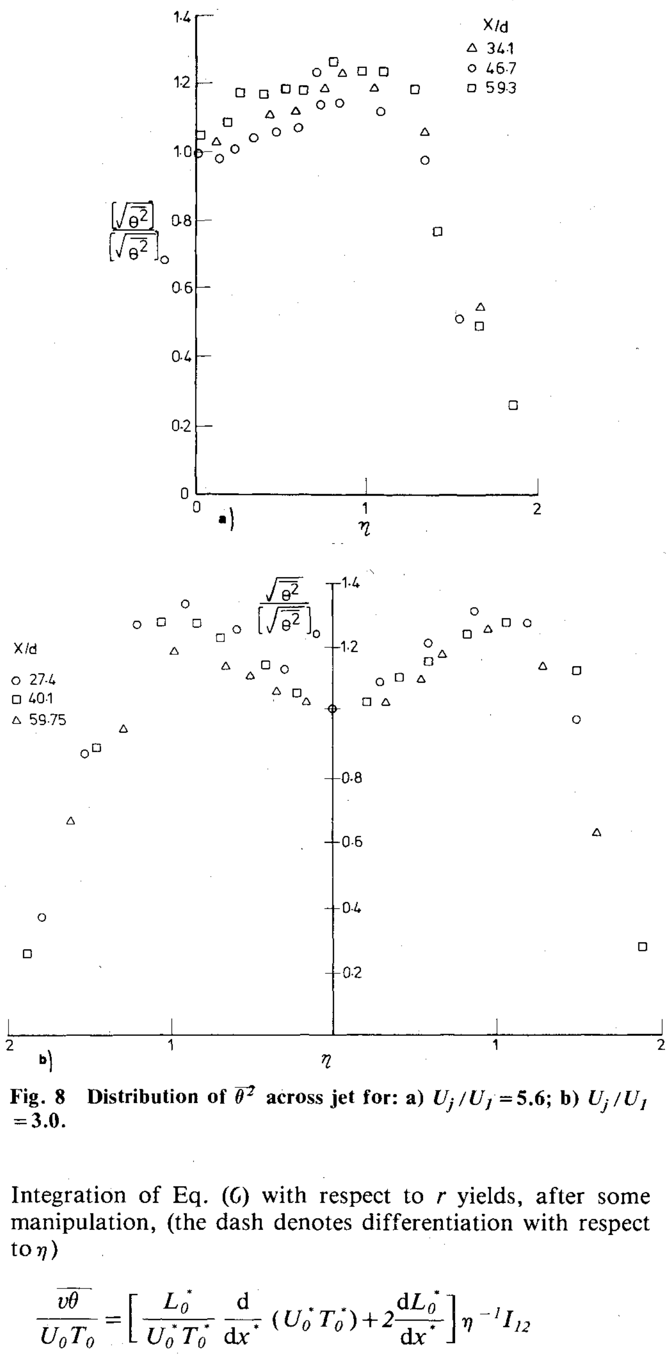

Fig. 8 Distribution of $\overline{\theta^2}$ across jet for: a) $U_j/U_l = 5.6$; b) $U_j/U_l = 3.0$.

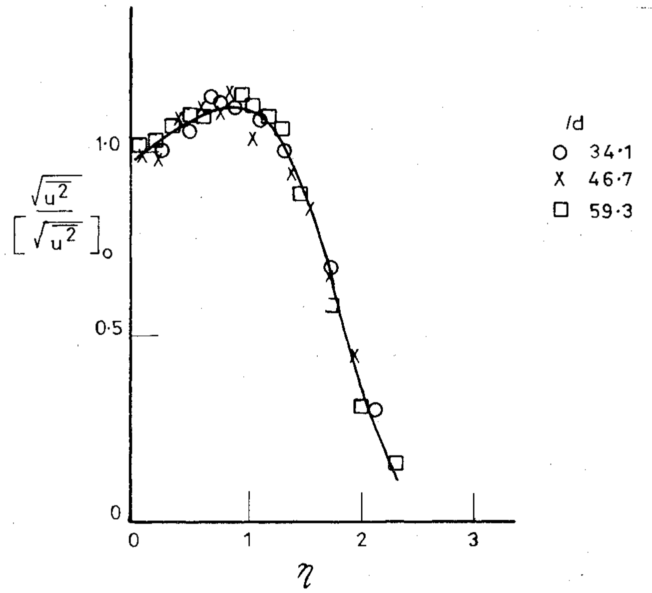

Fig. 9 Distribution of $\overline{u^2}$ across jet.

Integration of Eq. (6) with respect to r yields, after some manipulation, (the dash denotes differentiation with respect to η)

$$\frac{\overline{v\theta}}{U_0T_0} = \left[\frac{L_0^*}{U_0^*T_0^*}\frac{\mathrm{d}}{\mathrm{d}x^*}(U_0^*T_0^*) + 2\frac{\mathrm{d}L_0^*}{\mathrm{d}x^*}\right]\eta^{-1}I_{12} + \left(\frac{L_0^*}{U_0^*T_0^*}\frac{\mathrm{d}T_0^*}{\mathrm{d}x^*} + \frac{2}{U_0^*}\frac{\mathrm{d}L_0^*}{\mathrm{d}x^*}\right)\eta^{-1}I_2 - \left(\frac{L_0^*}{U_0^*}\frac{\mathrm{d}U_0^*}{\mathrm{d}x^*} + 2\frac{\mathrm{d}L_0^*}{\mathrm{d}x^*}\right)\eta^{-1}gI_1 - \frac{1}{U_0^*}\frac{\mathrm{d}L_0^*}{\mathrm{d}x^*}\eta g - \frac{\sigma^{-1}}{R}g' \qquad (9)$$

where

$$I_2 = \int_0^\eta \eta g \mathrm{d}\eta \qquad I_{12} = \int_0^\eta \eta f g \mathrm{d}\eta$$

R is the Reynolds number U_0L_0/ν, and σ is the Prandtl number ν/α. The asterisked nondimensional quantities are $L_0^* = L_0/d$, $U_0^* = U_0/U_l$, $x^* = x/d$, and $T_0^* = T_0/T_j$. Using the results of Figs. 3 and 4, distributions of $\overline{v\theta}/U_0T_0$ have been calculated at $x/d = 59$ for $U_j/U_l = 5.6$ and 3.0, and these are given in Fig. 10. The magnitude of $\overline{v\theta}/U_0T_0$ increases with decreasing U_j/U_l, in agreement with the trend of the $(\overline{\theta^2})^{1/2}/T_0$ results, and the maximum value occurs near $\eta = 0.9$, also in agreement with the trend of $(\overline{\theta^2})^{1/2}$ profiles of Fig. 8. The distribution of $\overline{v\theta}/U_0T_0$ calculated by Corrsin and Uberoi[4] for $U_l = 0$ has a peak value of only 0.016. It should be noted that the contribution from the viscous term $\sigma^{-1}g'/R$ of Eq. (9) is negligible, and the contributions from the second and third terms on the right-hand side of Eq. (9) were also found to be small, particularly for $U_j/U_l = 3.0$. For the axisymmetric wake, Freymuth and Uberoi[7] obtained the simplified expression (in the notation of this paper) $\overline{v\theta}/U_0T_0 \sim 0.515\eta g$, which has a maximum value of 0.27, significantly higher than for the present results.

Measured values of $\overline{u\theta}/U_0T_0$ on the axis of the jets are shown in Fig. 11. Within the scatter of the data, there is no significant variation of $\overline{u\theta}/U_0T_0$ with x, but the magnitude of $\overline{u\theta}/[(\overline{u^2})^{1/2}(\overline{\theta^2})^{1/2}]$ increases appreciably as U_j/U_l decreases. This trend is supported by the measurement of $\overline{u\theta}$ by Antonia et al.,[13] obtained in a heated jet ($U_j/U_l = 6.6$) with the use of an x-wire/cold wire arrangement. It should also be noted that Corrsin and Uberoi's[4] experimental $\overline{u\theta}$ values tend, somewhat surprisingly, to the isotropic value of zero for x/d as small as 25.

The Reynolds shear stress $\overline{uv}$ has been calculated from the momentum equation

$$U\frac{\partial U}{\partial x} + V\frac{\partial V}{\partial r} = -r^{-1}\frac{\partial}{\partial r}(r\overline{uv}) + \nu\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) \qquad (10)$$

Integration of Eq. (10) with respect to r and using Eqs. (2) and (8) yields, after some manipulation,

$$-\frac{\overline{uv}}{U_0^2} = -\frac{1}{U_0^*}\frac{\mathrm{d}L_0^*}{\mathrm{d}x^*}\eta f + \left(\frac{2}{U_0^*}\frac{\mathrm{d}L_0^*}{\mathrm{d}x^*} + \frac{L_0^*}{U_0^{*2}}\frac{\mathrm{d}U_0^*}{\mathrm{d}x^*}\right)\eta^{-1}I_1 - \left(2\frac{\mathrm{d}L_0^*}{\mathrm{d}x^*} + \frac{L_0^*}{U_0^*} \times \frac{\mathrm{d}U_0^*}{\mathrm{d}x^*}\right)\eta^{-1}fI_1 + \left(2\frac{L_0^*}{U_0^*}\frac{\mathrm{d}U_0^*}{\mathrm{d}x^*} + 2\frac{\mathrm{d}L_0^*}{\mathrm{d}x^*}\right)\eta^{-1}I_3 - \frac{f'}{R} \qquad (11)$$

where

$$I_3 = \int_0^\eta f^2\eta \mathrm{d}\eta$$

The distributions of $\overline{uv}/U_0^2$ are plotted in Fig. 12 and show the same trend as the distributions of $\overline{v\theta}/U_0T_0$. The con-

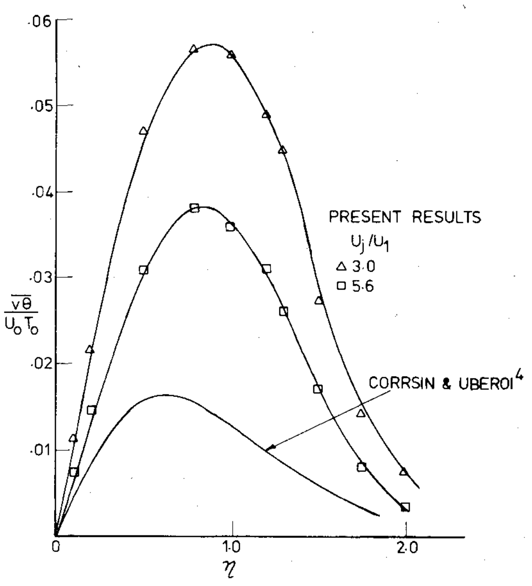

Fig. 10 Radial heat flux distribution across jet.

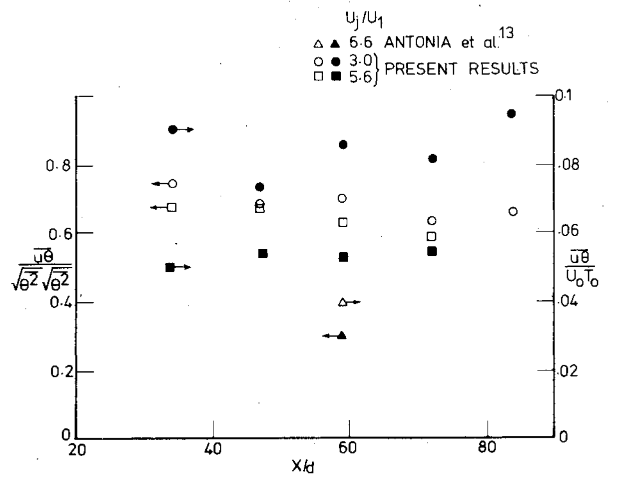

Fig. 11 Variation of correlation coefficient of u and θ (open symbols) on the centerline with axial distance. Solid symbols show $\overline{u\theta}$ normalized with U_0 and T_0.

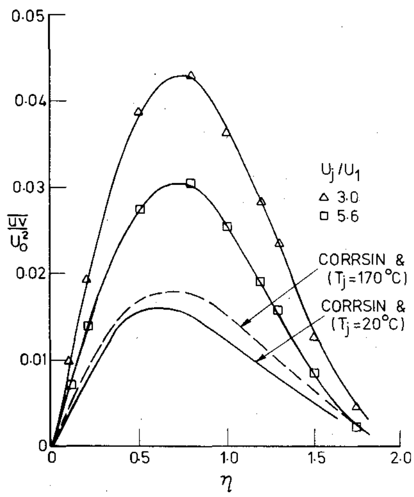

Fig. 12 Calculated Reynolds shear stress distributions across jet ($x/d = 59$).

tribution from the viscous term f'/R is negligible, while the major contributions to $\overline{uv}/U_0^2$ are from the first and fourth terms on the right-hand side of Eq. (11).

The turbulent Prandtl number σ_t, defined as the ratio $[-\overline{uv}/(\partial U/\partial r)]/[-\overline{v\theta}/(\partial T/\partial r)]$, may be written as

$$\sigma_t = \frac{\overline{uv}/U_0^2}{\overline{v\theta}/U_0T_0} \cdot \frac{g'}{f'} \qquad (12)$$

Using the calculated values of $\overline{uv}/U_0^2$ and $\overline{v\theta}/U_0T_0$, σ_t has been calculated for $U_j/U_1 = 5.6$ and 3.0.

The present results (Fig. 13.), together with the measured values of Antonia et al.[13] ($U_j/U_1 = 6.6$, $x/d = 59$), tend to indicate that in a jet with a coflowing stream there may be a small decrease in σ_t as U_j/U_1 decreases. This trend is not supported by the calculated values of Corrsin and Uberoi[4] or by values in the axisymmetric wake inferred from the data of Freymuth and Uberoi.[7,14] Although σ_t is approximately constant over a large part of the flow cross-section, the data in Fig. 13 do not conclusively indicate a variation of σ_t with U_j/U_1, probably as a result of the difficulty (see Ref. 15) of determining σ_t accurately.

The equation expressing the budget for $\overline{\theta^2}$ fluctuations across the jet can be written as

$$U\frac{\partial\overline{\theta^2}}{\partial x} + V\frac{\partial\overline{\theta^2}}{\partial r} + r^{-1}\frac{\partial}{\partial r}(r\overline{v\theta^2}) + 2\overline{v\theta}\frac{\partial\Delta T}{\partial r}$$
$$-\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\overline{\theta^2}}{\theta r}\right) + 6\alpha\left(\frac{\overline{\partial\theta}}{\partial x}\right)^2 = 0 \qquad (13)$$

The diffusion and dissipation terms have not been measured here, but the advection and production terms have been calculated from ΔT and $\overline{\theta^2}$ profiles across the jet. The advection term $(U\partial\overline{\theta^2}/\partial x + V\partial\overline{\theta^2}/\partial r)$, nondimensionalized by multiplying by $L_0/U_0T_0^2$, can be written as

$$\frac{V}{U_0}g_1' - \left(\frac{1+U_0^*f}{U_0^*}\right)\left(2\frac{L_0^*}{x^*}g_1 + g_1'\eta\frac{dL_0^*}{dx^*}\right)$$

where the similarity of the $\overline{\theta^2}$ profiles has been assumed with $\overline{\theta^2} = T_0^2 g_1(\eta)$. The nondimensional production term equal to $2(\overline{v\theta}/U_0T_0)g'$ is plotted in Fig. 14 together with the advection term. The advection increases with decreasing U_j/U_1 but at $U_j/U_1 = 3$ it is still lower than production except near the axis and the outer edge of the jet. In contrast, the advection measured by Freymuth and Uberoi[7] is one order of magnitude larger than the present advection and is also considerably larger than production over most of the jet cross-section.

V. Conclusions

The results confirm the view that the heated jet in an external stream lies within a spectrum of flows ranging from the heated jet into still air to the heated axisymmetric wake. Beyond about 20 diameters from the jet outlet, normalized mean and fluctuating temperature and velocity profiles show almost no influence of either axial distance or of velocity ratio U_j/U_1 and closely resemble those found for the jet into still air and axisymmetric heated wake. The trend to wake-like behavior is not so evident in the present work for T_0 and U_0, both of which vary as x^{-1}; however, the limited range of x/d should be noted and it could be expected that $x^{-2/3}$ behavior would be found at higher x/d as in the case of the isothermal jet.[1] The behavior of the jet temperature width, L_0, is consistent with this behavior of U_0 and T_0 and is found to be always greater than the velocity width L_u. This last result supports results for a jet into still air and a wake[8] but is in disagreement with Freymuth and Uberoi's[7] observation that $L_0 \approx L_u$ in the wake.

As was found for the isothermal jet, there is no universal similarity in spite of the similarity of normalized mean and

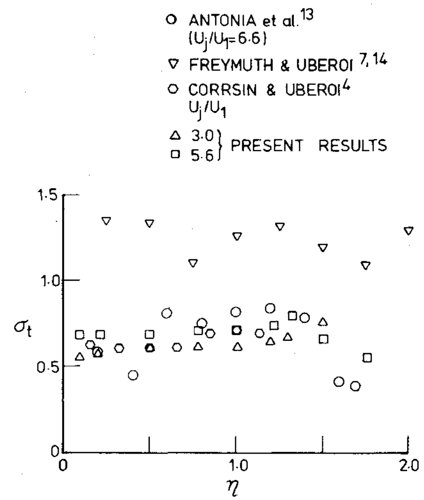

Fig. 13 Turbulent Prandtl number variation across a jet and a wake.

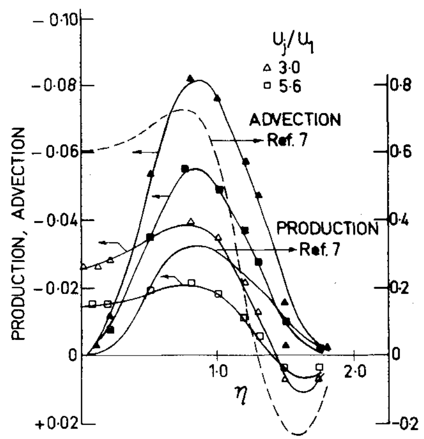

Fig. 14 Production and advection of temperature fluctuations across a jet and a wake.

fluctuating velocity and temperature profiles. The values of $(\overline{\theta^2})^{1/2}{}_0/T_0$ vary with x and with U_j/U_l, as was found for $(\overline{u^2})^{1/2}{}_0/U_0$ in isothermal flows and also in the present heated flow. For high values of U_j/U_l these scale ratios are very close to those found for the jet into still air, while at low values of U_j/U_l and large x they tend toward the much higher values found in wakes. The radial turbulent heat transfer $\overline{v\theta}$ $/U_0T_0$ is found to be much larger than that for the jet into still air but not as large as that for the axisymmetric wake. A similar trend is observed for the Reynolds shear stress $\overline{uv}/U_0^2$. Calculated values of the turbulent Prandtl number σ_t are, within the experimental scatter, approximately constant over most of the flow cross-section, but no definite conclusion on the variation of σ_t with velocity ratio U_j/U_l can be drawn from the existing data.

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